AC circuits: inductive reactance, capacitive reactance, impedance and admittance

- voltage-current profiles of L, C, RL, RC and circuits
- impedance of RL, RC and RLC circuits

Notes from Gibilisco; Teach Yourself Electricity and Electronics, McGraw-Hill, Chapters 13-15
Inductive reactance \((X_L)\)

Inductance = property of circuit where change in electric current through the circuit induces an electromotive force that opposes the change in current = ......

\[
X_L = 6.28 fL
\]

Increase AC frequency – what will be the effect?
Examples

• inductance = 0.50 H, frequency = 60 Hz, what is $X_L$?

• for the same circuit as above, we use a battery with 12 VDC, what is $X_L$?

What are the units of $X_L$?
Voltage-current profile

For pure **inductance** circuit

Current lags driving (source) voltage

For pure **resistance** circuit

Current in phase with driving (source) voltage

13-8  In a pure inductance, the current lags the voltage by 90 degrees.

13-11 In a circuit with only resistance, the current is in phase with the voltage.
Inductive reactance and resistance

13-10 Schematic representation of an RL circuit.

13-9 In a circuit with inductance and resistance, the current lags the voltage by less than 90 degrees.
Combined inductance and resistance

13-7 Four vectors in the RL impedance plane.
Phase angle

Apply Pythagoras to obtain hypotenuse = impedance

\[ \sin \phi = \]
Capacitive reactance \( (X_C) \)

Similarly to inductance, capacitance has an effect on impedance.

By convention, capacitive reactance is expressed as a negative ohmic value.

![Diagram showing capacitive and inductive reactance on a number line.](image-url)
AC and capacitance

What is the voltage-current profile going to look like?
Current leads voltage

Capacitive reactance:

\[ X_C = -\frac{1}{(6.28 fC)} \]
Inverse relationship

Intuitive understanding:
Larger C $\Rightarrow$ more movement of charge $\Rightarrow$ smaller resistance
Higher f $\Rightarrow$ more movement of charge $\Rightarrow$ approach short-circuit
Examples

• capacitor = 0.001 \( \mu \text{F} \), frequency = 1 MHz, what is \( X_C \)?

• for the same circuit as above, we use a battery with 12 VDC, what is \( X_C \)?

For the second question, what this mean for transmission on DC across a capacitor?
Combined capacitive reactance and resistance

14-9 In a circuit with capacitance and resistance, the current leads the voltage by less than 90 degrees.

Work out phase angle

14-7 Four vectors in the RC impedance plane.
Impedance (Z) = resistance to AC current, composed of resistance, inductive reactance and capacitive reactance.
Examples:

• $R = 50 \, \Omega$, $X_L = 22 \, \Omega$, $X_C = -33 \, \Omega$

• $R = 50 \, \Omega$, $X_L = 444 \, \Omega$, $X_C = -444 \, \Omega$

$Z = R + j(X_L + X_C)$
Inverse definitions

DC circuit

Resistance ($R$) $\neq$ conductance ($G$)

AC circuit

Impedance ($Z$) $\neq$ admittance ($Y$)

Reactance ($X$) $\neq$ susceptance ($B$)

**BUT**, note sign inversion for imaginary numbers:

$B_C = 1/X_C$, but due to imaginary numbers:

$jB_C = -j/X_C$

Example:

$X_C = 10 \, \Omega$, what is $jB_C$?

$jB_C = 1/(j10) = (1/j)(1/10) = -j \, 0.1$
RLC in-parallel

\[ Y = G + j(B_L + B_C) \]

Example:
- \( G = 0.1 \) Siemens, \( B_L = -j0.010 \),
- \( B_C = j0.020 \)